write the first 3 nonzero terms & the general term

1. $e^x \quad 1+x+\frac{x^2}{2}+\dots+\frac{x^n}{n!}$

2. $(\sin x)^2 = \left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+\dots+\frac{x^{2n+1}}{(2n+1)!}(-1)^n\right)^2$

3. $\cos x \quad 1-\frac{x^2}{2}+\frac{x^4}{4!}\dots+\frac{x^{2n}}{(2n)!}(-1)^n$

4. $\ln(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}\dots+\frac{x^n}{n}(-1)^{n+1}$

5. $\frac{1}{1-x} = 1+x+x^2+\dots+x^n$

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23. $e^x = 1+x+\frac{x^2}{2}$

$|x|<0.1$

$-0.1 \le x \le 0.1$

$\text{error} \le \left|\frac{M\, x^{n+1}}{(n+1)!}\right|$

$n=2$

$\left|\frac{M\,x^3}{3!}\right|$

$M = \text{Max of } f''' = e^x$

$M = e^{0.1}$

$\text{error} \le \left|\frac{e^{0.1}(0.1)^3}{3!}\right| = .000184$

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9. $\sin^2 x = \frac{1-\cos(2x)}{2}$ p565

$\cos x = 1-\frac{x^2}{2}+\frac{x^4}{4!}\dots$

$\cos(2x) = 1-\frac{(2x)^2}{2}+\frac{(2x)^4}{4!}\dots$

$1-\cos(2x) = 1-\left(1-\frac{(2x)^2}{2}+\frac{(2x)^4}{4!}\dots\right)$

$\frac{1-\cos(2x)}{2} = \frac{(2x)^2}{2\cdot 2}-\frac{(2x)^4}{2\cdot 4!}+\frac{(2x)^6}{2\cdot 6!}\dots\frac{(2x)^{2n}(-1)^{n+1}}{2\cdot(2n)!}$

$n=1 \quad 2 \quad 3$

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9.4a Tests for Convergence of Series

nth term test for divergence

If the seq. does not converge to 0 then the series diverges

If the seq converges to 0, the series might converge (test fails)

Does the series converge or diverge?

$\sum_{n=1}^{\infty}\frac{n+1}{n} = 2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}\dots$

$= 1+1+1+1\dots$

seq. converges to 1

series diverges

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The direct comparison test.

If $0 \le a_n \le b_n$ & $\sum b_n$ converges, then $\sum a_n$ converges

If $a_n \ge b_n \ge 0$ & $\sum b_n$ diverges, then $\sum a_n$ diverges

Does the series converge or diverge?

$$\sum_{n=0}^{\infty} \frac{3^n}{5^n+1} \le \sum_{n=0}^{\infty} \frac{3^n}{5^n} = 1 + \frac{3}{5} + \frac{9}{25} + \frac{27}{125} \cdots$$

converges by comparison

geo $r = \frac{3}{5} < 1$

converges

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Absolute convergence

If $\sum |a_n|$ converges than $\sum a_n$ converges

Show the series converges for all x

$$\sum_{n=0}^{\infty} \frac{(\sin(x))^n}{n!}$$

converges absolutely

$$\sum_{n=0}^{\infty} \left|\frac{(\sin(x))^n}{n!}\right| \le \sum_{n=0}^{\infty} \frac{1}{n!} = e$$

converges by comparison

$$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} \cdots$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots$$

let $x = 1$

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The Ratio Test

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$$

if $L < 1$, $\sum a_n$ converges

if $L > 1$, $\sum a_n$ diverges

if $L = 1$ test fails

Does the series converge or diverge?

$$\sum_{n=0}^{\infty} \frac{3^n}{n!}$$ converges

$$\lim_{n\to\infty} \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} = \lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n}$$

$$\frac{n!}{(n+1)!} = \frac{\cancel{n!}}{(n+1)\cancel{n!}}$$

$$= \lim_{n\to\infty} \frac{3}{n+1} = 0 = L$$

$L < 1$

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do 9.4: 29-45 all

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